CONFIDENTIAL

UNIVERSITY COLLEGE TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET

COURSE CODE	: BGE 1133
COURSE	: CALCULUS
SEMESTER/SESSION	:1-2022/2023
DURATION	: 3 HOURS

Instructions:

1. This booklet contains 5 questions in SECTION A, 3 questions in SECTION B and 2 questions in SECTION C. Answer ALL questions.
2. All answers should be written in the answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Differentiate the following functions with respect to x.

a) $f(x)=\dfrac{2}{4x^3}-20x^3+25x^2-\dfrac{7}{x}$ (2 marks)

b) $f(x)=\sqrt[3]{x^5}+\cos 5x-\ln x-e^{-2x}$ (4 marks)

c) $f(x)=4e^{-x}+\left(e^x+e^{-x}\right)\left(e^x-e^{-x}\right)$ (4 marks)

QUESTION 2

Evaluate the following integrals.

a) $\int\left(5x^4+2x^2-2x+7\right)dx$ (2 marks)

b) $\int\left(\cos 3x-e^{2x}+\dfrac{1}{2x}+\sqrt{x}\right)dx$ (4 marks)

c) $\int\left(\dfrac{8}{\sqrt{x}}+\sin 2x+e^{\frac{3}{2}x+1}+\dfrac{1}{7x}\right)dx$ (4 marks)

QUESTION 3

Find the derivatives for the following using the given techniques.

a) $f(x)=\left(3x^2-1\right)\ln|2x-1|$ (Use Product Rule) (3 marks)

b) $f(x)=\dfrac{5e^{3x}}{\cos 2x}$ (Use Quotient Rule) (4 marks)

c) $f(x)=\sqrt[3]{e^{2x}+x^2}$ (Use Chain Rule) (3 marks)

QUESTION 4

Integrate the following integrals using the given techniques.

a) $\int \frac{\ln 2x}{x^5}\,dx$ (Use integration by Part method) (5 marks)

b) $\int \frac{x^2+1}{x^3+3x}\,dx$ (Use substitution method) (5 marks)

c) $\int \frac{6x}{(x-2)(x+1)}\,dx$ (Use Partial Fractions method) (5 marks)

QUESTION 5

Find the inverse Laplace transform of

$$\frac{4}{s^2-9}+\frac{3}{(s+2)^2}$$

(5 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

a) Find the general solution of the differential equation below.

$$y^2 \frac{dy}{dx} = x^2 - 1$$ (4 marks)

b) Solve the differential equation using the integrating factor method.

$$x\frac{dy}{dx} - y = 10x^2$$ (6 marks)

QUESTION 2

a) Solve the differential equation $y'' - 2y' + 3y = 0$ (5 marks)

b) Find the particular solution of the differential equation that satisfies the given conditions.

$$y'' - y' - 2y = 0 \quad ; \ y(0) = 2 \ , \ y'(0) = 1$$ (10 marks)

QUESTION 3

Find the area enclosed by the curve $y^2 = x - 3$ and the y-axis, $y = -1$ and $y = 2$. Hence sketch the graph. (5 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Given $y = x(x-3)(x+5)$. Find:

a) the coordinates of stationary points. (5 marks)

b) the local extreme point. (3 marks)

c) hence sketch the graph. (2 marks)

QUESTION 2

Find the particular solution of the differential equation by using the method of Laplace Transform.

$$y' + 4y = 12e^{5t}; \qquad y(0) = 2$$ (10 marks)

.....................................END OF QUESTIONS.....................................

FORMULA	
$\frac{d}{dx}(uv) = uv' + vu'$	$\int \sin f(x)\, dx = -\frac{1}{f'(x)} \cos f(x) + C$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$	$\frac{d}{dx} e^x = e^x$
$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} g'(x)$	$\frac{d}{dx} \ln x = \frac{1}{x}$
$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$	$\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)$
$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$	$\frac{d}{dx}\left(\ln\lvert f(x)\rvert\right) = \frac{1}{f(x)} \cdot f'(x)$
$\frac{d}{dx} \sin x = \cos x$	$\frac{d}{dx} \sin f(x) = f'(x) \cos f(x)$
$\frac{d}{dx} \cos x = -\sin x$	$\frac{d}{dx} \cos f(x) = -f'(x) \sin f(x)$
$yV(x) = \int q(x) V(x) dx$	$\int e^x\, dx = e^x + C$
$V(x) = e^{\int p(x) dx}$	$\int e^{ax+b}\, dx = \frac{1}{a} e^{ax+b} + C$
$y - y_1 = m(x - x_1)$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$y - y_1 = -\frac{1}{m}(x - x_1)$	$\int \sin x\, dx = -\cos x + C$
$\int u\, dv = uv - \int v du$	$\int \cos x\, dx = \sin x + C$
$\int f[g(x)] g'(x) dx = \int f(u) du$	$\int \cos(ax + b)\, dx = \frac{1}{a} \sin(ax + b) + C$
$\int_0^\infty e^{-st} f(t) dt$	$Area = \int_a^b [f(x) - g(x)] dx$

TABLE OF TRIAL PARTICULAR SOLUTION , y_p

No.	$g(x)$	y_p
1.	1	A
2.	$5x+7$	$Ax+B$
3.	$\sin 2x$	$A\cos 2x + B\sin 2x$
4.	$\cos 2x$	$A\cos 2x + B\sin 2x$
5.	e^{5x}	Ae^{5x}

TABLE OF LAPLACE TRANSFORMS

No.	$y(t)$	$\mathcal{L}\{y(t)\} = F(s)$
1.	1	$\frac{1}{s}$
2.	t	$\frac{1}{s^2}$
3.	t^n	$\frac{n!}{s^{n+1}}$, $n=1,2,\ldots$
4.	e^{at}	$\frac{1}{s-a}$
5.	$\sin at$	$\frac{a}{s^2+a^2}$
6.	$\cos at$	$\frac{s}{s^2+a^2}$
7.	y'	$s\mathcal{L}\{y\} - y(0)$
8.	y''	$s^2\mathcal{L}\{y\} - sy(0) - y'(0)$